**MATHEMATICS**

**GRADE 10**

**MATRICES 2**

Identity Matrix and Solving Simultaneous Equation Matrix Method

IDENTITY MATRIX

The identity matrix is$ I=\left[\begin{matrix}1&0\\0&1\end{matrix}\right]$. It is a square matrix with ones in the leading diagonal and zeroes in the non-leading diagonal. When a matrix is multiplied by its inverse the product is the identity matrix.

Example 1:Find the inverse of matrix A

$$A=\left[\begin{matrix}2&1\\-1&5\end{matrix}\right]$$

If you recall the inverse of matrix A $\left(A^{-1}\right)$ can be calculated by $\frac{1}{detA}×A $Adjoint. The determinant of A is $\left(2×5\right)-\left(1×-1\right)=10--1=11. $The adjoint of matrix A is $\left[\begin{matrix}5&-1\\1&2\end{matrix}\right]$.

$$A^{-1}=\frac{1}{11}\left[\begin{matrix}5&-1\\1&2\end{matrix}\right]$$

Applying scalar multiplication we multiply each element in the adjoint matrix by $\frac{1}{11}. $

$$A^{-1}=\left[\begin{matrix}\frac{1}{11}×5&\frac{1}{11}×-1\\\frac{1}{11}×1&\frac{1}{11}×2\end{matrix}\right]$$

After which you multiply to obtain the final matrix: $A^{-1}\left[\begin{matrix}\frac{5}{11}&-\frac{1}{11}\\\frac{1}{11}&\frac{2}{11}\end{matrix}\right]$.

A MATRIX MULTIPLIED BY ITS INVERSE

$$A^{-1}A orAA^{-1}=I$$

$A=\left[\begin{matrix}2&1\\-1&5\end{matrix}\right]×\left[\begin{matrix}\frac{5}{11}&-\frac{1}{11}\\\frac{1}{11}&\frac{2}{11}\end{matrix}\right]$=$\left[\begin{matrix}2×\frac{5}{11}+1×\frac{1}{11}&2×-\frac{1}{11}+1×\frac{2}{11}\\-1×\frac{5}{11}+5×\frac{1}{11}&-1×-\frac{1}{11}+5×\frac{2}{11}\end{matrix}\right]$

Simplifying: $\left[\begin{matrix}\frac{10}{11}+\frac{1}{11}&-\frac{2}{11}+\frac{2}{11}\\-\frac{5}{11}+\frac{5}{11}&\frac{1}{11}+\frac{10}{11}\end{matrix}\right]$=$\left[\begin{matrix}\frac{11}{11}&\frac{0}{11}\\\frac{0}{11}&\frac{11}{11}\end{matrix}\right]$=$\left[\begin{matrix}1&0\\0&1\end{matrix}\right]$.

Any matrix multiplied by the identity matrix will result in the original matrix.

$\left[\begin{matrix}1&0\\0&1\end{matrix}\right]×\left[\begin{matrix}2&-3\\1&4\end{matrix}\right]$=$\left[\begin{matrix}1×2+0×1&1×-3+0×4\\0×2+1×1&0×-3+1×4\end{matrix}\right]=$

$$\left[\begin{matrix}2+0&-3+0\\0+1&0+4\end{matrix}\right]=\left[\begin{matrix}2&-3\\1&4\end{matrix}\right]$$

SIMULTANEOUS EQUATION (MATRIX METHOD)

Solve $5x-2y=16 and 7x+6y=-4$

1. Write the coefficients in the equations presented in a 2x2 matrix called A, express the variables in a column matrix called X and the solutions in a column matrix called B. This forms the matrix equation expressed below:

$$AX=B$$

$$\left[\begin{matrix}5&-2\\7&6\end{matrix}\right]×\left[\begin{matrix}x\\y\end{matrix}\right]=\left[\begin{matrix}16\\-4\end{matrix}\right]$$

1. To find X, you pre-multiply both sides by$ A^{-1}$, hence $A^{-1}AX=A^{-1}B. $This will result $IX=A^{-1}B$. Any matrix multiplies by the identity matrix result in the original matrix, $X=A^{-1}$B.

$$\left[\begin{matrix}x\\y\end{matrix}\right]=\frac{1}{30--14}\left[\begin{matrix}6&2\\-7&5\end{matrix}\right]\left[\begin{matrix}16\\-4\end{matrix}\right]$$

$$\left[\begin{matrix}x\\y\end{matrix}\right]=\frac{1}{44}\left[\begin{matrix}6&2\\-7&5\end{matrix}\right]\left[\begin{matrix}16\\-4\end{matrix}\right]$$

Apply scalar multiplication to get:

$$\left[\begin{matrix}x\\y\end{matrix}\right]=\left[\begin{matrix}\frac{3}{22}&\frac{1}{22}\\-\frac{7}{44}&\frac{5}{44}\end{matrix}\right]\left[\begin{matrix}16\\-4\end{matrix}\right]$$

$$\left[\begin{matrix}x\\y\end{matrix}\right]=\left[\begin{matrix}\frac{3}{22}×16+\frac{1}{22}×-4\\-\frac{7}{44}×16+\frac{5}{44}×-4\end{matrix}\right]$$

Simplify: $\left[\begin{matrix}x\\y\end{matrix}\right]=\left[\begin{matrix}\frac{24}{11}+-\frac{2}{11}\\-\frac{28}{11}+ -\frac{5}{11}\end{matrix}\right]=\left[\begin{matrix}\frac{22}{11}\\-\frac{33}{11}\end{matrix}\right]=\left[\begin{matrix}2\\-3\end{matrix}\right]$

**Therefore:** $x=2 and y=-3$